

1. The only integer pair with a difference equal to the quotient is (4, 2). Find the sum of the only pair of positive fractions with denominator 8 in lowest terms with this property.
- a. 11.0                      b. 11.25                      c. 11.5                      d. 11.75                      e. 12.0

The property can be represented by:  $a - b = \frac{a}{b}$

Solving the equation for a:  $a - b = \frac{a}{b} \rightarrow ab - b^2 = a \rightarrow a(b - 1) = b^2 \rightarrow a = \frac{b^2}{b-1}$

Let  $b = \frac{c}{8}$  to represent one of the positive fractions with denominator 8 in lowest terms.

Substituting into  $a = \frac{b^2}{b-1}$  gives  $a = \frac{\frac{c^2}{64}}{\frac{c}{8}-1} \rightarrow a = \frac{c^2}{8c-64}$

Since a is a fraction with denominator 8,  $8c - 64 = 8 \rightarrow c = 9$

Substituting  $c = 9$  gives  $a = \frac{81}{8}$  and  $b = \frac{9}{8}$  and the sum  $a + b = 11.25$

2. The interior angles of an n-gon form an arithmetic sequence with the first term  $128^\circ$  and common difference  $4^\circ$ . There are two values of n which satisfy this condition. Find their sum.
- a. 20                      b. 21                      c. 22                      d. 25                      e. 27

The sum of the interior angles of an n-gon is  $180(n-2)$ .

The arithmetic sequence of interior angles of the n-gon is represented by:

$$128 + 132 + 136 + \dots + (128 + 4(n - 1))$$

The condition is satisfied when

$$128 + 132 + 136 + \dots + (128 + 4(n - 1)) = 180(n - 2)$$

Using the formula for the sum of an arithmetic sequence, this equation becomes

$$\frac{n}{2}(128 + (128 + 4(n - 1))) = 180(n - 2)$$

Which can be simplified to the quadratic equation

$$n^2 - 27n + 180 = 0$$

And the solutions are  $n = 12$  and  $n = 15$ , so the sum is 27.

3. For the function  $f(x)$ ,  $f(1) = 4$ . Also,  $f(x) \cdot f(y) = f(x+y) + f(x-y)$  for all real numbers  $x$  and  $y$ . Find  $f(5)$ .

a. 720

b. 724

c. 728

d. 732

e. 736

Using the property  $f(x) \cdot f(y) = f(x+y) + f(x-y)$ , let  $x = 1$  and  $y = 0$

$$f(1) \cdot f(0) = f(1) + f(1)$$

Since  $f(1) = 4$ ,  $f(1) \cdot f(0) = f(1) + f(1) \rightarrow 4 \cdot f(0) = 8 \rightarrow f(0) = 2$

Now, using the property  $f(x) \cdot f(y) = f(x+y) + f(x-y)$ , let  $x = 1$  and  $y = 1$

$$f(1) \cdot f(1) = f(2) + f(0)$$

$$4 \cdot 4 = f(2) + 2$$

$$f(2) = 14$$

Now, using the property  $f(x) \cdot f(y) = f(x+y) + f(x-y)$ , let  $x = 2$  and  $y = 1$

$$f(2) \cdot f(1) = f(3) + f(1)$$

$$14 \cdot 4 = f(3) + 4$$

$$f(3) = 52$$

Now, using the property  $f(x) \cdot f(y) = f(x+y) + f(x-y)$ , let  $x = 3$  and  $y = 2$

$$f(3) \cdot f(2) = f(5) + f(1)$$

$$52 \cdot 14 = f(5) + 4$$

$$f(5) = 724$$